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Equilibrium computation for two-player games in strategic and extensive form

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We give an introduction to the structure of Nash equilibria of bimatrix games based on best-reply regions derived from the payoff matrices. With this geometric insight, one can construct games with certain properties, and understand algorithms for computing equilibria. We explain the classic Lemke-Howson algorithm that finds one Nash equilibrium, which also shows that a generic game has an odd number of equilibria.

We also present a very canonical way of understanding the INDEX of an equilibrium, which is important for dynamic and strategic stability, by showing that the two equilibria that are the endpoints of a Lemke-Howson path have opposite index. Time premitting we present essentially the full proof, and thus provide a streamlined version of the following paper: Lloyd Shapley (1974), A note on the Lemke-Howson algorithm. Mathematical Programming Study 1: Pivoting and Extensions, 175-189.

The second part of the tutorial describes compact strategies for game trees. We consider game trees with imperfect information, also called extensive games. We present a new strategic description for game trees, called the sequence form. Its central idea is the use of move sequences rather than strategies (complete contingency plans), which brings about an exponential improvement in size over the conventional strategic form. The main use of the sequence form is in algorithms for finding Nash equilibria of the game tree. For a two-player game, a particularly simple pivoting algorithm due to Lemke, similar to the simplex algorithm for linear programming, finds one sample equilibrium, and mimicks the "tracing procedure" by Harsanyi and Selten.

The relevant concepts and issues relating to algorithms will be introduced and discussed for non-experts.